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MA222 - Computational Linear Algebra
Problem Sheet - 5

Basic Ideas from Linear Algebra and Vector Norms

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1. Show that if $A \in \mathbb{R}^{m\times n}$ has rank p, then there exists an $X \in \mathbb{R}^{m\times n}$ and a $Y \in \mathbb{R}^{n\times p}$ such that $A = XY^T$, where $rank(X) = rank(Y) = p$.

2. Suppose $A(\alpha) \in \mathbb{R}^{m\times r}$ and $B(\alpha) \in \mathbb{R}^{r\times n}$ are matrices whose entries are differentiable functions of the scalar a. Show
$$\frac{d}{d\alpha}[A(\alpha)B(\alpha)] = \left[\frac{d}{d\alpha}A(\alpha)\right]B(\alpha) + A(\alpha)\left[\frac{d}{d\alpha}B(\alpha)\right].$$

3. Suppose $A(\alpha) \in \mathbb{R}^{n\times n}$ has entries that are differentiable functions of the scalar α. Assuming $A(\alpha)$ is always nonsingular, show
$$\frac{d}{d\alpha}[A(\alpha)^{-1}] = -A(\alpha)^{-1}\left[\frac{d}{d\alpha}A(\alpha)\right]A(\alpha)^{-1}.$$

4. Suppose $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$ and that $\phi(x) = \frac{1}{2}x^TAx - x^Tb$. Show that the gradient of ϕ is given by $\nabla\phi(x) = \frac{1}{2}(A^T + A)x - b$.

5. Assume that both A and $A + uv^T$ are nonsingular where $A \in \mathbb{R}^{n\times n}$ and $u, v \in \mathbb{R}$. Show that if x solves $(A + uv^T)x = b$, then it also solves a perturbed right hand side problem of the form $Ax = b + \alpha x$. Give an expression for α in terms of A, u, and v.

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